

Radiating Shear-Free Gravitational Collapse with Charge

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Abstract

We present here a new shear free model for the gravitational collapse of a spherically symmetric charged body. We propose a dissipative contraction with radiation emitted outwards represented by the Vaidya-Reissner-Nordström metric. The Einstein field equations, using the junction conditions and an ansatz, are integrated numerically. A check of the energy conditions is also performed. We obtain that the charge delays the Reissner-Nordström black hole formation and it can even prevent the collapse.

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I. INTRODUCTION

The equilibrium of the stars depends on the balance of two conflicting effects. The gravitational force attracting the material of the star toward the center, and the opposite internal thermal pressure provided by nuclear fusion of the elements in the stellar interior. When these reactions cease and no other source of pressure acts significantly, like the non-thermal degeneracy pressure, this balance is broken and a massive star undergoes the continuous catastrophic contraction, the gravitational collapse. The remnant product of this process can be a black hole.

The General Theory of Relativity furnishes the foundation for analyzing the effects of the huge gravitational field involved in this process. In this context, the first idealized model for the gravitational collapse was proposed by Oppenheimer and Snyder [1], in 1939. It consists on the dynamics of a simple pressureless fluid (dust), a set of no interacting particles subjected only to the action of the gravitational field generated by their own masses. In consequence, the particles travel along timelike geodesics towards the center of the configuration, resulting in the formation of a Schwarzschild black hole. Thus, an interesting step forward is to verify the consequences in considering more relevant fluids, for instance, endowed with pressure, viscosity and/or electric charge. Since this landmark paper, many attempts aiming a more realistic description of this phenomenon have been taken place, but this usually impose a more complete set of equations, and the non linearity coming from the own structure of the field equations often compel us to the application of numerical procedures.

An interesting issue lies on the fact that, if a star could hold a non null amount of net electric charge, the contraction of such an object could give rise to the Reissner-Nordström black hole. However, the existence of considerable fraction of net charge in the stellar interior has been for long rejected. A process proposed by Rosseland [2] asserts that the free electrons of the strongly ionized gas that composes the star can be ejected due to its large thermal velocities. As the balance of forces is established, the process is interrupted and an object containing only 100 Coulomb per solar mass remains. Similar arguments supported by Glendenning [3] reaffirm the neutrality of the stars. However, these objections

are grounded in the Newtonian context and, as it was already advertised [4], do not apply when extremely huge gravitational fields are in play. Indeed, in the scenario of the Theory of the General Relativity, a relativistic star could hold greater portions of charge keeping itself stable ([4], [5]), albeit no mechanism producing charge asymmetry is known. A work in this direction can be found in [6]. Nevertheless, these issues seem far from being concluded, making it important to draw attention to the charged stellar models.

There exist several relativistic static interior charged fluid solutions connected to the exterior Reissner-Nordström spacetime. A good classification and description of most of them can be found in Ivanov, 2002 [7]. Among these solutions, the one developed by Cooperstock and De La Cruz [8] will be useful for our investigation. It represents a charged dust fluid, where the T_{00} component of the stress-energy tensor in the comoving frame – composed by the matter and the interior electric field – is a constant, giving rise to a Schwarzschild-like interior solution.

In spite of the several static charged interior metrics, dynamic solutions are rare. In this sense, an interesting work performed by Bekenstein in 1971 [9] has indicated the dual role played by the charge. Depending on the charge-mass ratio, the electric field can contribute to the support or, in opposition, to the speeding up of the structure's collapse (charge regeneration). The proof of the Birkhoff theorem and the hydrostatic equilibrium equation for the charged case were also achieved.

The Vaidya-Reissner-Nordström metric, generalization of the Vaidya's metric [10] in the presence of electromagnetic field, enables the geometrical description of the radiation field surrounding the star, allowing the development of several models of radiative gravitational collapse of charged stars [11–14]. Among these models, the non adiabatic shear free model of de Oliveira and Santos [11] is very similar to the one proposed in the present paper, despite the fact that the field equations have not been integrated before and of the absence of pressure anisotropy. Some years earlier, the same authors and Kolassis [15] studied the uncharged case by means of the separation of the metric functions of the general spherically symmetric isotropic interior metric into distinct functions of the radial and temporal coordinates, a proposition already known from a preceding investigation [16]. Years later these models have been generalized by Chan for the shearing model [17]. This simplification allows the complete integration of the Einstein field equations, provided a radial function representing the static pre-collapse configuration is given. Here we use this same assumption, besides the

selection of the referred Cooperstock and De La Cruz solution [8] for the the radial function that represents the initially static situation. We are going to see that this ansatz forces the physical variables out of equilibrium to have the same radial behavior as if they were in equilibrium. This is called in the literature as the post-quasi-static approximation [18–20].

In studying a static relativistic stellar model, Bowers & Liang [21] pointed to the relevant effects played by the anisotropy of pressure over the maximum mass for a stable system and over the surface redshift. among other factors, pressure anisotropy is induced by the electric charge [22], by the shear movement [17, 23] and, moreover, it is enlarged by the shear viscosity [24] (see [25] for a detailed review article about the occurrence of local pressure anisotropy in the study of self-gravitating systems). The shear movement, on the other hand, affects the luminosity and total collapse time of the contracting systems [26, 27], justifying its inclusion in several works. However, in an unsuccessful attempt to obtain an exact solutions of the Einstein's equations, in this paper we forsake the general case of shearing collapse, although, due to others sources, the pressure anisotropy is taken into account.

Here we have to remind important consequences of the shear-free condition already studied. It was shown by Herrera and Santos [28, 29] that, in the quasi-static approximation and for the non-dissipative situation, the shear-free flow is equivalent to the homologous contraction in the Newtonian limit (details about homologous contraction, homology relations and its application in the astrophysics can be found in [30]). Moreover, considering dissipation in both regimes, diffusion and streaming out, the authors verified the need to impose homology conditions on the temperature and emission rate to keep the homologous evolution [28]. In addition, it was checked that the requirement of homogeneous expansion implies homology conditions. Another important aspect is that the shear-free condition is not assured all along the collapse of spheres whether the dissipative processes, the density inhomogeneity and the pressure anisotropy are taking in account [31]. Hence, this follows, leastwise in the study of geodesic motion of the fluid.

Current literature is comprised by some comprehensive models of charged collapsing radiating spheres with shear motion. A good example is the work developed by Medina and collaborators [12] with the aid of a method developed by Herrera et. al. [32]. Following the procedure, a system of first order differential equations evaluated at the surface is obtained and integrated. The same method was applied by Rosales et. al. [14] in more recent paper. Treating the same subject, but using the approach of Misner and Sharp [33], Di Prisco and

collaborators [34] analyzed the dynamics of the charged spheres.

The stellar contraction must be a highly dissipative process due mainly to the interaction of the radiation and matter in the interior, such that, in the same way we are considering emission of energy across the stellar surface, an outward flux of energy through the body occurs. The energy transport is often considered in both regimes, diffusion and free-streaming. Once the studies of the radiation of the type-II supernova 1987A pointed to the domain of the former [35], this is the only one taken into account in this work. Details on how the energy flux happens and how it relates to the temperature are not the focus here. This could be performed, at first glance, with the usage of the relativistic thermodynamic theory developed by Eckart [36]. But, although simple, this theory has an undesirable feature, the equations for the heat transfer and viscous tensions indicate instantaneous propagation of the perturbations, generating causality problems. This difficulty was overcome with the Israel-Stewart theory [37], where the relaxation times for the dissipative quantities were included, resulting in hyperbolic equations for the perturbations (a complete discussion can be found in [38]). The energy transport equations of this theory supplied the thermodynamic analysis in several works. In an interesting paper, Di Prisco and collaborators [39] compared the luminosity profile of the collapsing spheres for the vanishing (Eckart) and non-vanishing (Israel-Stewart) relaxation times. The same idea was used in a different model by the same authors and some more collaborators [40] one year later. In this way, some effects of the pre-relaxation process were confirmed to be model-independent (more on the influence of the relaxation time in the outcome of the collapsing systems in [41]). In applying the transport equation to collapsing sphere models, the authors usually simplify the problem making null the constants coupling heat and viscosity. Few years ago, Herrera and collaborators [42], using the the method by Misner & Sharp [33], studied the dissipative gravitational collapse when this simplification is set aside. In this way, the authors have appreciated the influence of these coupling constants over the effective inertial mass of the system.

The aim of the present investigation is to find a dynamical solution for a charged sphere representing the dissipative collapse of the charged body, as well as to verify the role played by the electric charge in this process. The formation or avoidance of the black hole is discussed. The paper is organized as follows, in Section II we set the metrics and the stress-energy tensor used, we show the field equations and obtain a set of equations by the application of the junction conditions. The solution of the system of equations is performed

in Section III. The equations for the static initial configuration are presented in Section IV. The results can be found in Section V and the agreement with the energy conditions are checked the Section VI. Finally, we discuss the results in Section VII.

II. EQUATIONS

The system is composed by an interior spacetime (referred by a minus signal "-"), a comoving timelike hypersurface (referred by Σ) and an exterior spacetime representing a radiation field surrounding the body (referred by a plus sign "+"). The unknown interior spacetime is given by the spatially isotropic spherically symmetric shear-free metric in comoving coordinates,

$$ds_-^2 = -A^2(r, t)dt^2 + B^2(r, t) [dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)]. \quad (1)$$

The exterior spacetime is described by Vaidya-Reissner-Nordström metric, which represents an outgoing radial flux of radiation around a charged spherically symmetric source of gravitational field, given by

$$ds_+^2 = - \left[1 - \frac{2m(v)}{r} + \frac{Q^2}{r^2} \right] dv^2 - 2dvdr + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (2)$$

where $m(v)$ represents the mass, function of the retarded time v , and Q is the total amount of charge of the system inside the boundary surface Σ .

The interior stress-energy tensor of a charged dissipative anisotropic fluid is given by

$$T_{\alpha\beta} = (\mu + p_\perp)u_\alpha u_\beta + p_\perp g_{\alpha\beta} + (p - p_\perp)X_\alpha X_\beta + q_\alpha u_\beta + q_\beta u_\alpha + \frac{1}{4\pi} \left[F_\alpha^\gamma F_{\beta\gamma} - \frac{1}{4} g_{\alpha\beta} F^{\gamma\delta} F_{\gamma\delta} \right]. \quad (3)$$

These quantities are the energy density μ , the radial pressure p , the tangential pressure p_\perp , the radial heat flux q^α , u^α is the four-velocity and X^α is a radial four-vector. The following relations hold: $u_\alpha q^\alpha = 0$, $X_\alpha X^\alpha = 1$, $X_\alpha u^\alpha = 0$. The tensor $F_{\alpha\beta}$ represents the electromagnetic field tensor. At last, the coupling constant in geometrized units is $\kappa = 8\pi$ (i.e., $c = G = 1$).

As we use comoving coordinates, we have

$$u^\alpha = A^{-1} \delta_0^\alpha, \quad (4)$$

and as the heat flux is radial

$$q^\alpha = q\delta_1^\alpha. \quad (5)$$

The Maxwell's equation are written as

$$F_{\alpha\beta} = \phi_{\beta,\alpha} - \phi_{\alpha,\beta}, \quad (6)$$

and

$$F^{\alpha\beta}_{;\beta} = 4\pi J^\alpha, \quad (7)$$

where ϕ_α is the four-potential and J^α is the four-current. It is assumed that the charge is at rest with respect to the coordinates of the metric (1), so we have no magnetic field present and, thus, we can write

$$\phi_\alpha = \Phi\delta_\alpha^0, \quad (8)$$

and

$$J^\alpha = \sigma u^\alpha. \quad (9)$$

Where Φ is the scalar potential and σ is the charge density.

From equations (8) and (6) we obtain

$$F_{01} = -F_{10} = -\frac{\partial\Phi}{\partial r}. \quad (10)$$

Substituting equations (9) and (10) into Maxwell's equations (6) and (7), we have

$$\Phi'' + \left(-\frac{A'}{A} + \frac{B'}{B} + \frac{2}{r}\right)\Phi' = 4\pi AB^2\sigma, \quad (11)$$

and

$$\frac{\partial}{\partial t}\left(\frac{\Phi'}{A^2B^2}\right) + \left(-\frac{\dot{A}}{A} + 3\frac{\dot{B}}{B}\right)\frac{\Phi'}{A^2B^2} = 0, \quad (12)$$

where the dot represents differentiation with respect to t and the prime with respect to r .

Integrating these equations, we obtain

$$\Phi' = \frac{A}{B} \frac{l}{r^2}, \quad (13)$$

where $l(r)$ is the radial distribution of charge given by

$$l(r) = 4\pi \int_0^r \sigma B^3 r^2 dr. \quad (14)$$

The integration from the center ($r = 0$) till the surface ($r = r_\Sigma$) provides us the total amount of charge Q inside the body.

The non-vanishing components of the field equations applied to the interior spacetime are very similar to the paper [11], except by the presence – justified in the first section – of the anisotropy of pressure. These are obtained with the aid of the equations (1), (3), (4), (5), resulting in

$$G_{00}^- = -\left(\frac{A}{B}\right)^2 \left[2\frac{B''}{B} - \left(\frac{B'}{B}\right)^2 + \frac{4}{r}\frac{B'}{B} \right] + 3\left(\frac{\dot{B}}{B}\right)^2 = \kappa \left(A^2\mu + \frac{1}{8\pi} \frac{A^2}{B^4} \frac{l^2}{r^4} \right), \quad (15)$$

$$\begin{aligned} G_{11}^- &= \left(\frac{B'}{B}\right)^2 + \frac{2}{r}\frac{B'}{B} + 2\frac{A'}{A}\frac{B'}{B} + \frac{2}{r}\frac{A'}{A} + \left(\frac{B}{A}\right)^2 \left[-2\frac{\ddot{B}}{B} - \left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} \right] \\ &= \kappa B^2 \left(p - \frac{1}{8\pi} \frac{1}{B^4} \frac{l^2}{r^4} \right), \end{aligned} \quad (16)$$

$$\begin{aligned} G_{22}^- &= \frac{1}{\sin^2\theta} G_{33}^- = \left[\frac{B''}{B} - \left(\frac{B'}{B}\right)^2 + \frac{1}{r}\frac{B'}{B} + \frac{A''}{A} + \frac{1}{r}\frac{A'}{A} \right] \\ &\quad + \frac{B^2}{A^2} \left[-2\frac{\ddot{B}}{B} - \left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} \right] = \kappa B^2 \left(p_\perp + \frac{1}{8\pi} \frac{1}{B^4} \frac{l^2}{r^4} \right), \end{aligned} \quad (17)$$

$$G_{01}^- = -2\frac{\dot{B}'}{B} + 2\frac{B'}{B}\frac{\dot{B}}{B} + 2\frac{A'}{A}\frac{\dot{B}}{B} = -\kappa AB^2 q. \quad (18)$$

We can note that the equations presented in this section are a particular case of those present in the section II of the paper [34]. Here we are assuming no shearing motion and we neglect the shear viscosity as well as the dissipation in the free-streaming regime.

We consider a spherical surface with its motion described by a timelike three-surface Σ , which splits spacetimes into interior and exterior manifolds. The connection between these geometries has to be continuous and smooth, assured by the junction conditions. Here we follow the approach given by [43], [44] and demand the continuity of the metric $(ds_-^2)_\Sigma = (ds_+^2)_\Sigma = ds_\Sigma^2$ and of the the extrinsic curvature $K_{ij}^- = K_{ij}^+$ across the surface Σ (details on the junction conditions used here can be seen in the paper [11]).

The condition of continuity of the metric leads us to

$$\frac{dt}{d\tau} = A(r_\Sigma, t)^{-1}, \quad (19)$$

$$r_{\Sigma}B(r_{\Sigma}, t) = \mathbf{r}_{\Sigma}(v), \quad (20)$$

and

$$\left(\frac{dv}{d\tau}\right)_{\Sigma}^{-2} = \left(1 - \frac{2m}{\mathbf{r}} + 2\frac{d\mathbf{r}}{dv} + \frac{Q^2}{\mathbf{r}^2}\right)_{\Sigma}, \quad (21)$$

where τ is a proper time defined on the comoving hypersurface Σ .

The non-vanishing extrinsic curvature components are $K_{\tau\tau}^{\pm}$, $K_{\theta\theta}^{\pm}$ e $K_{\phi\phi}^{\pm}$.

Equating $K_{\theta\theta}^{-}$ and $K_{\theta\theta}^{+}$ we have

$$\left[\left(\frac{dv}{d\tau}\right)\left(1 - \frac{2m}{\mathbf{r}} - \frac{Q^2}{\mathbf{r}^2}\right)\mathbf{r} + \frac{d\mathbf{r}}{d\tau}\mathbf{r}\right]_{\Sigma} = [r(rB' + B)]_{\Sigma}. \quad (22)$$

With the help of equations (19), (20), (21), we can write equation (22) as

$$m = \left[\frac{r^3 B \dot{B}^2}{2A^2} - r^2 B' - \frac{r^3 B'^2}{2B} + \frac{Q^2}{2rB}\right]_{\Sigma}, \quad (23)$$

which is the total energy entrapped inside the surface Σ [45].

Equating $K_{\tau\tau}^{-}$ and $K_{\tau\tau}^{+}$, using equation (19), we have

$$\left[\frac{d^2 v}{d\tau^2} \left(\frac{dv}{d\tau}\right)^{-1} - \left(\frac{dv}{d\tau}\right) \frac{m}{\mathbf{r}^2} + \frac{Q^2}{\mathbf{r}^3} \frac{dv}{d\tau}\right]_{\Sigma} = -\left(\frac{A'}{AB}\right)_{\Sigma}. \quad (24)$$

Substituting equations (19), (20) and (23) into (22) we can write

$$\left(\frac{dv}{d\tau}\right)_{\Sigma} = \left[\frac{(rB)'}{B} + \frac{(rB)'}{A}\right]_{\Sigma}^{-1}. \quad (25)$$

This is the gravitational redshift for the observer at rest at the infinity. Its divergence indicates the formation of an event horizon. This happens when the factor in parentheses goes to zero.

Substituting the derivative of this last equation and equation (23) into (24), identifying the geometrical terms with the field equations (16) and (18), we have

$$p_{\Sigma} = (qB)_{\Sigma}. \quad (26)$$

This result is the same of the one obtained by de Oliveira et. al. [15] and years later in the study of charged spheres [11] and [13].

We can use the equations (23), (19), (25) and (16) to obtain the total luminosity for an observer at rest at infinity

$$L_{\infty} = -\left(\frac{dm}{dv}\right)_{\Sigma} = \frac{\kappa}{2} \left[pr^2 B^2 \left(1 + \frac{rB'}{B} + \frac{r\dot{B}}{A}\right)^2\right]_{\Sigma}. \quad (27)$$

III. SOLUTION OF THE FIELD EQUATIONS

In order to integrate the field equations, we resort to the same ansatz referred in the Section I, consisting in separate the metric functions $A(r, t)$ and $B(r, t)$ into functions of the coordinates r and t in the form

$$A(r, t) = A_0(r), \quad (28)$$

and

$$B(r, t) = B_0(r)f(t), \quad (29)$$

where $A_0(r)$ and $B_0(r)$ are solutions of a static charged dust fluid.

We have chosen this separation of variables in the metric functions in such way that when $f(t \rightarrow -\infty) \rightarrow 1$, the metric functions represent the initial static solution. The decreasing (or increasing) of $f(t)$ provides the decreasing (or increasing) of the physical radius ($R = rB(r, t)$) of the body, describing the collapse (or expansion). The process ends when $f(t \rightarrow 0) \rightarrow 0$.

Now the equations (15)-(18) can be written as

$$\kappa\mu = \kappa\frac{\mu_0}{f^2} + \frac{l^2}{r^4 B_0^4 f^2} \left(1 - \frac{1}{f^2}\right) + \frac{3}{A_0^2} \left(\frac{\dot{f}}{f}\right)^2, \quad (30)$$

$$\kappa p = \kappa\frac{p_0}{f^2} - \frac{l^2}{r^4 B_0^4 f^2} \left(1 - \frac{1}{f^2}\right) + \frac{1}{A_0^2} \left[-2\frac{\ddot{f}}{f} - \left(\frac{\dot{f}}{f}\right)^2\right], \quad (31)$$

$$\kappa p_\perp = \kappa\frac{p_{0\perp}}{f^2} + \frac{l^2}{r^4 B_0^4 f^2} \left(1 - \frac{1}{f^2}\right) + \frac{1}{A_0^2} \left[-2\frac{\ddot{f}}{f} - \left(\frac{\dot{f}}{f}\right)^2\right], \quad (32)$$

$$\kappa q = -\frac{2A'_0}{A_0^2 B_0^2} \frac{\dot{f}}{f^3}, \quad (33)$$

where

$$\kappa\mu_0 = -\frac{1}{B_0^2} \left[2\frac{B_0''}{B_0} - \left(\frac{B_0'}{B_0}\right)^2 + \frac{4}{r}\frac{B_0'}{B_0}\right] - \frac{l^2}{r^4 B_0^4}, \quad (34)$$

$$\kappa p_0 = \frac{1}{B_0^2} \left[\left(\frac{B_0'}{B_0}\right)^2 + \frac{2}{r}\frac{B_0'}{B_0} + 2\frac{A'_0}{A_0}\frac{B_0'}{B_0} + \frac{2}{r}\frac{A'_0}{A_0}\right] + \frac{l^2}{r^4 B_0^4}, \quad (35)$$

and

$$\kappa p_{0\perp} = \frac{1}{B_0^2} \left[\frac{B_0''}{B_0} - \left(\frac{B_0'}{B_0} \right)^2 + \frac{1}{r} \frac{B_0'}{B_0} + \frac{A_0''}{A_0} + \frac{1}{r} \frac{A_0'}{A_0} - \frac{l^2}{r^4 B_0^2} \right], \quad (36)$$

Substituting equations (31), (33) into (26) and assuming also that $p_0(r_\Sigma) = 0$, we obtain a second order differential equation for $f(t)$,

$$2f^3 \ddot{f} + f^2 \dot{f}^2 - 2\bar{a} f^2 \dot{f} - \bar{b} (1 - f^2) = 0, \quad (37)$$

where

$$\bar{a} = \left(\frac{A_0'}{B_0} \right)_\Sigma, \quad (38)$$

and

$$\bar{b} = \left(\frac{A_0^2 Q^2}{r^4 B_0^4} \right)_\Sigma. \quad (39)$$

This equation is similar to the one obtained by de Oliveira et al. in 1985 [15], except by the last term. The attempts to solve it analytically proved to be unfortunate, at last, the numerical solution was successful. In order to verify the electric field action, different values of total charge were considered.

IV. MODEL OF THE INITIAL CONFIGURATION

We consider that the system at the beginning of the collapse has a static configuration of a charged dust fluid described in Cooperstock e De La Cruz [8]. The Schwarzschild-like interior solution is obtained considering

$$T_0^0 = \lambda + \frac{E^2}{8\pi} = \frac{3a^2}{8\pi}, \quad (40)$$

where T_0^0 is the mass-energy density component of the stress-energy tensor, λ is the mass density, E is the electric field and a is a constant.

The metric for this case, presented here in isotropic coordinates, is given by

$$ds_-^2 = A_0^2(\bar{r}) dt^2 + B_0^2(\bar{r}) [d\bar{r}^2 + \bar{r}^2(d\theta^2 + \sin^2 \theta d\phi^2)], \quad (41)$$

where

$$A_0^2(\bar{r}) = \frac{\beta^2}{4} (1 + \bar{r}^2)^2, \quad (42)$$

$$B_0^2(\bar{r}) = \frac{4}{a^2 C^2 (1 + \bar{r}^2)^2}, \quad (43)$$

$$\bar{r} = r/C, \quad (44)$$

and the constants β and a are given by

$$\beta = 1 - a^2 R^2 + (1 - a^2 R^2)^{1/2},$$

$$a^2 = \frac{2m_0}{R^3} - \frac{Q^2}{R^4}.$$

The quantities m_0 , $R = (r_\Sigma B_0(r_\Sigma))$ and Q are the total mass, radius and charge of the object. The constant C comes from the integration of the original Schwarzschild coordinates transformed into the isotropic coordinates. It can be easily determined by junction conditions.

The coefficients (38) and (39) become

$$\begin{aligned} \bar{a} &= \frac{a^2 \beta R}{(1 + \sqrt{1 - a^2 R^2})^2} \\ \bar{b} &= \frac{\beta^2 Q^2}{R^4 (1 + \sqrt{1 - a^2 R^2})^2}. \end{aligned} \quad (45)$$

Substituting the functions $A_0(\bar{r})$, $B_0(\bar{r})$, their derivatives and a rewritten version of the equations (37) into the equations (30) - (35), we obtain

$$\kappa\mu = \frac{\kappa\mu_0}{f^2} + \frac{l^2 a^4}{16\bar{r}^4 f^2} (1 + \bar{r}^2)^4 \left(1 - \frac{1}{f^2}\right) + \frac{12}{\beta^2 f^2 (1 + \bar{r}^2)^2} \dot{f}^2, \quad (46)$$

$$\kappa p = -\frac{l^2 a^4}{16\bar{r}^4 f^2} (1 + \bar{r}^2)^4 \left(1 - \frac{1}{f^2}\right) + \frac{4}{\beta^2 f^2 (1 + \bar{r}^2)^2} \left[\bar{b} \left(1 - \frac{1}{f^2}\right) - 2\bar{a}\dot{f} \right], \quad (47)$$

$$\kappa p_\perp = \frac{l^2 a^4}{16\bar{r}^4 f^2} (1 + \bar{r}^2)^4 \left(1 - \frac{1}{f^2}\right) + \frac{4}{\beta^2 f^2 (1 + \bar{r}^2)^2} \left[\bar{b} \left(1 - \frac{1}{f^2}\right) - 2\bar{a}\dot{f} \right], \quad (48)$$

$$\kappa q = -\frac{2a^2 C \bar{r}}{\beta} \frac{\dot{f}}{f^3}, \quad (49)$$

where

$$\begin{aligned} \mu_0(\bar{r}) &= \frac{a^2 (3 - \bar{r}^2)}{8\pi}, \\ p_0(\bar{r}) &= p_{\perp 0}(\bar{r}) = 0. \end{aligned} \quad (50)$$

The two last equations correspond to the charged dust solution of the paper [8], but in isotropic coordinates. For the charge distribution we have

$$l^2(\bar{r}) = \frac{16\bar{r}^6}{a^2(1 + \bar{r}^2)^4}. \quad (51)$$

V. RESULTS

The integration of the differential equation (37) depends on the coefficients \bar{a} and \bar{b} (equation (45)), related to the values of mass, radius and charge of the object before the beginning of the collapse. We have taken $m_0 = 5M_\odot$, $R = 15$ km and different values for the ratio charge-mass $Q/m_0 \simeq 0.00$, $Q/m_0 = 0.20, 0.40, 0.64$. The result is shown in Figure 1, from which we notice that the collapse is slower with the increasing of the charge. This result is in contrast to that obtained by Medina et al. [12], pointed as unexpected by the authors.

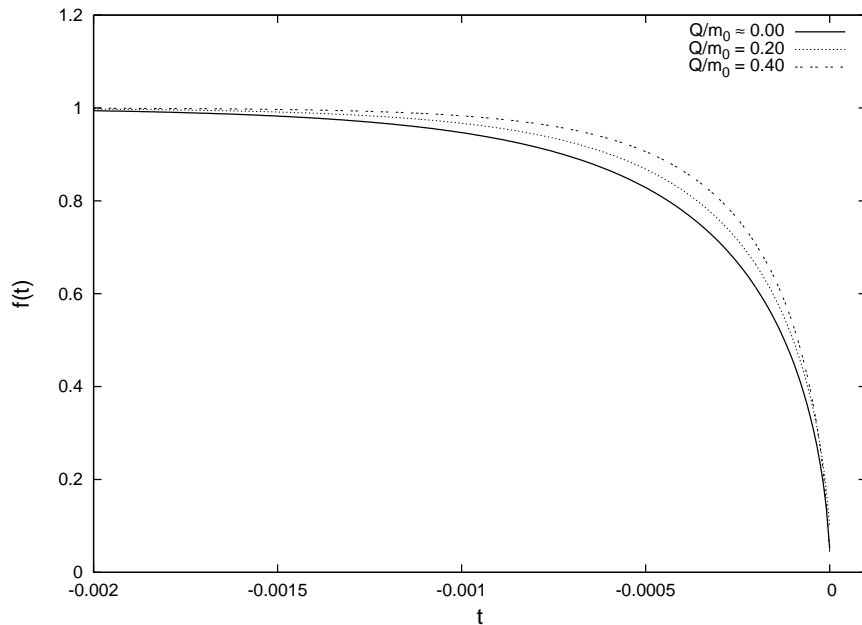


FIG. 1: Temporal behavior of the function $f(t)$ for the models with $Q/m_0 \simeq 0.0, 0.20, 0.40$. The time is in seconds.

In Section II, we declared that the formation of an event horizon occurs when the surface gravitational redshift goes to infinity, i.e., the term in the parentheses in the equation (25) goes to zero. Using equations (28), (29) and the metric functions (42) and (43) into the

parentheses of that equation, we have

$$\left(1 + \frac{rB'_0}{B_0} + \frac{rB_0\dot{f}}{A_0}\right)_\Sigma = \left[1 - \frac{a^2 R^2}{1 + \sqrt{1 - a^2 R^2}} + \frac{R(1 + \sqrt{1 - a^2 R^2})}{\beta} \dot{f}_{bh}\right] = 0. \quad (52)$$

So, the event horizon arises when

$$\dot{f}_{bh} = \frac{-\beta^2}{R(1 + \sqrt{1 - a^2 R^2})^2}. \quad (53)$$

Note from Figure 2 that, when the amount of charge is not very large, the event horizon is always formed, however, for larger values, this condition is no longer satisfied.

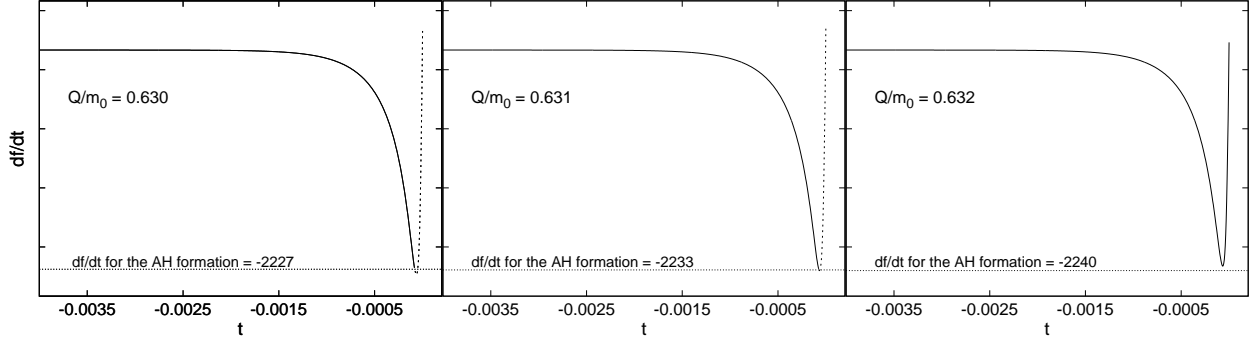


FIG. 2: Gravitational redshift condition for a star with initial mass $5 M_\odot$ and initial radius $R = 15$ km. For $Q/m_0 > 0.6310$, the function \dot{f} no longer reaches the value for the event horizon formation.

In the last stages of the contraction of the overly charged stars ($Q/m_0 > 0.6310$), the decreasing of the function $f(t)$ decelerates and it reaches a minimum value (see the Figure 3). Beyond this point, the function increases sharply and a bounce of the system seems to occur. This idea is false, since the luminosity becomes negative for positive \dot{f} (see equation (55) below). This is physically inconsistent, and we have to stop the integration at this instant. The solution is just valid until this minimum point, where the luminosity, heat flux and the rate of decrease of the stellar radius (\dot{f}) go to zero. Nevertheless, the mass is reduced along the contraction by the radiative emission, but some portion still lasts at the endpoint. The same is truth for the stellar radius. This suggests that an equilibrium situation could be the final fate of the evolution.

The equations (23) and (27) become

$$m = \frac{R^3}{2\beta^2} \left(1 + \sqrt{1 - a^2 R^2}\right) f \dot{f}^2 + \frac{a^2 R^3}{2} f + \frac{Q^2}{2Rf}, \quad (54)$$

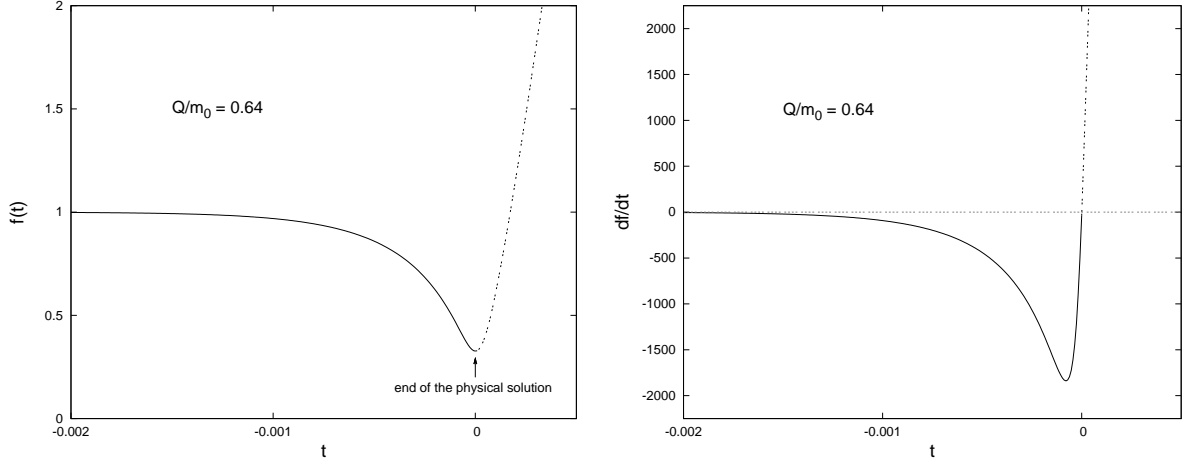


FIG. 3: Time evolution of $f(t)$ for a highly charged star.

$$L = -\bar{a}R^2\dot{f} \left[1 + \frac{R^2}{\beta^2} (2\beta + a^2R^2) \dot{f} \right]^2. \quad (55)$$

Note that, before the collapse, when $f(t) \rightarrow 1$ and $\dot{f} \rightarrow 0$, the equation (54) becomes

$$m_0 = \frac{a^2R^3}{2} + \frac{Q^2}{2R}, \quad (56)$$

as we expected from [8].

The Figure 4 shows the temporal evolution of the mass and luminosity. We can note that models with larger values of charge favor higher emission of energy. However, in the case where we have no black hole formation, this situation is reversed, as we can see from the right bottom panel.

The integrated physical quantities of the fluid are plotted in the figures 5, 6, 7 and 8. Due to the fact that the charge distribution is an increasing function of r , for the charged case, the radial pressure in the outermost radial coordinates are greater than the pressure in the innermost.

VI. ENERGY CONDITIONS FOR THE CHARGED FLUID

In principle, any geometry might represent, via Einstein's equations, a valid solution of the theory of General Relativity, once nothing is inferred about the matter content of the source of the gravitational field, or, precisely, no restrictions are set on the components of the stress-energy tensor. On the other hand, the known forms of matter are endowed with

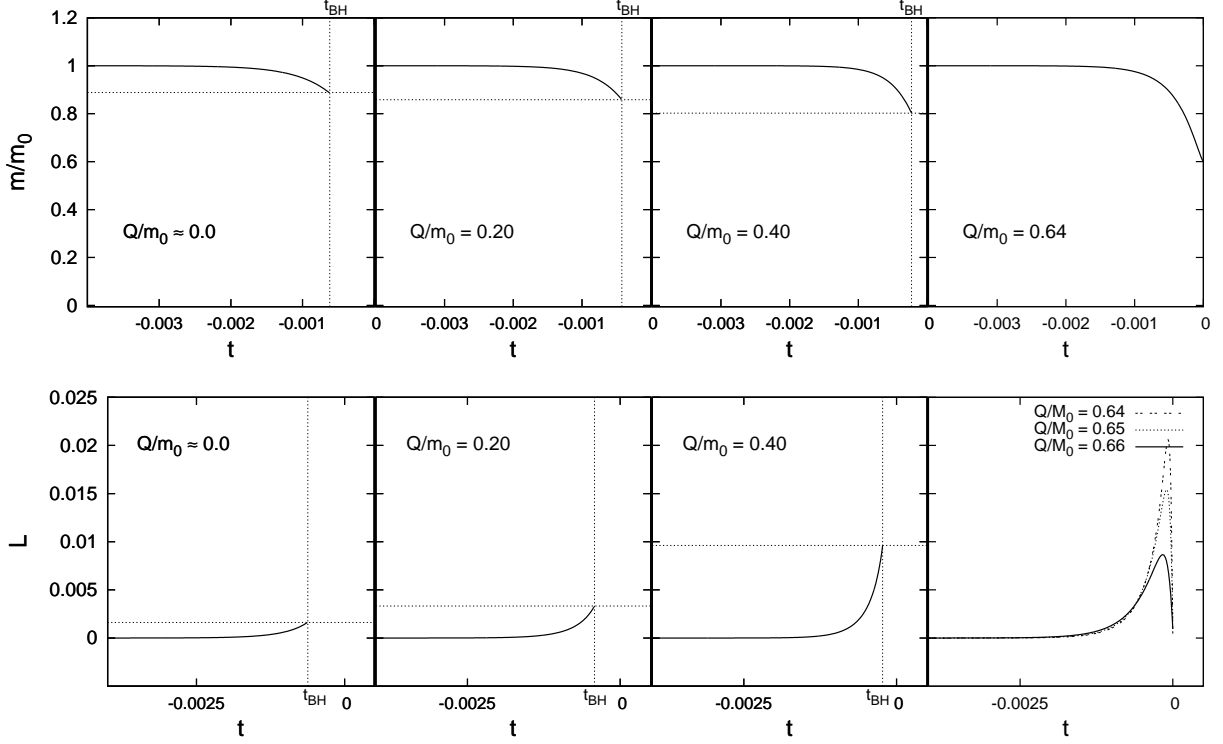


FIG. 4: Mass and luminosity for different charge/initial mass ratio models in geometric units.

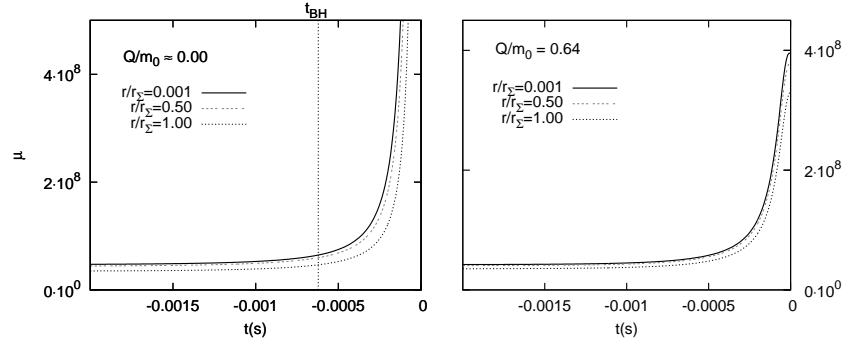


FIG. 5: Temporal evolution of the density for an almost uncharged and a highly charged star in geometric units.

certain physical features, like positivity of the energy density or like its domain over the principal stresses, for example. Desired physically meaningful solutions, these features are combined in a set of constraints, the energy conditions, guiding the acceptance or disposal of a particular solution of the field equations.

In order to trace the energy conditions along the moment of contraction of the body, we follow the same procedure used in Kolassis, Santos & Tsoubelis [46]. Here we skip the

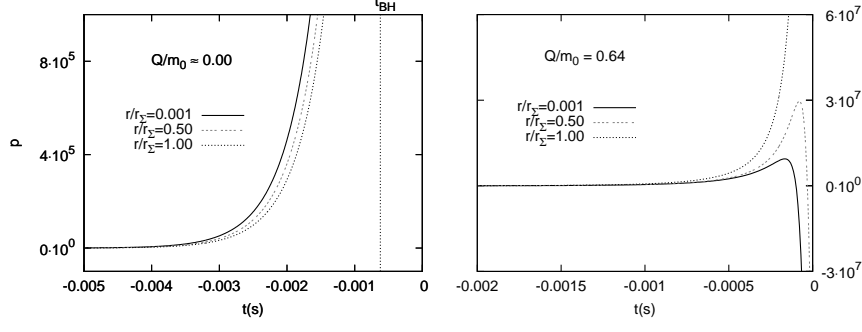


FIG. 6: Temporal evolution of the pressure for an almost uncharged and a highly charged star in geometric units.

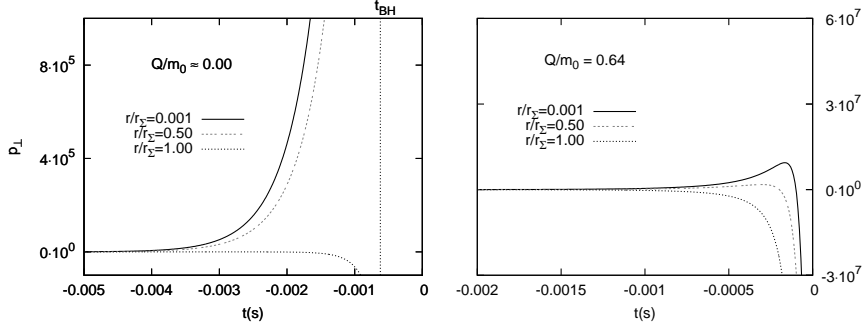


FIG. 7: Temporal evolution of the tangential pressure for an almost uncharged and a highly charged star in geometric units.

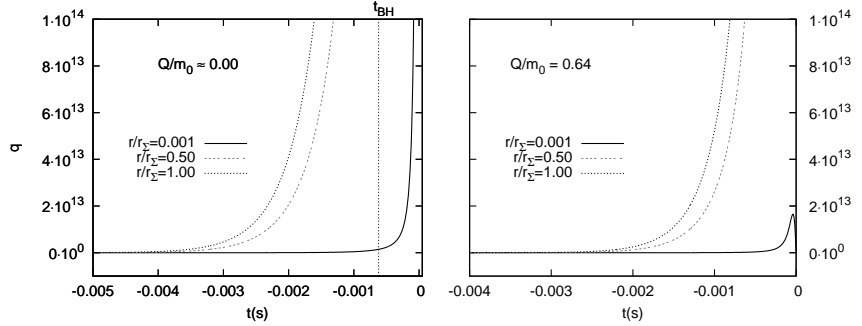


FIG. 8: Temporal evolution of the heat flux for an almost uncharged and a highly charged star in geometric units.

mathematical step by step, warning that the similar approach is found in [23, 47]. These conditions for the charged fluid in question are fulfilled if the following inequalities are satisfied:

Mathematical condition

$$(i) \quad \sqrt{\Delta} \geq 0$$

Common to all the energy conditions

$$(ii) \quad \frac{1}{2} \left[\mu + p + \frac{1}{8\pi} \frac{l^2(r)}{r^4 B^4} \left(\frac{1}{B^2} + 2 \right) + \sqrt{\Delta} \right] \geq 0$$

Weak conditions:

$$(iii) \quad \frac{1}{2} \left[\mu - p + \frac{1}{8\pi} \frac{l^2(r)}{r^4 B^6} + \sqrt{\Delta} \right] \geq 0$$

Dominant conditions:

$$(iv) \quad \frac{1}{2} \left[\mu - 3p + \frac{1}{8\pi} \frac{l^2(r)}{r^4 B^4} \left(\frac{1}{B^2} - 2 \right) + \sqrt{\Delta} \right] \geq 0$$

$$(v) \quad \frac{1}{2} \left[\mu - p + \frac{1}{8\pi} \frac{l^2(r)}{r^4 B^6} \right] \geq 0$$

Strong conditions:

$$(vi) \quad \sqrt{\Delta} + 2p + \frac{1}{4\pi} \frac{l^2(r)}{r^4 B^4} \geq 0.$$

Where

$$\Delta = \left[(\mu + p) - \frac{1}{8\pi} \frac{l^2(r)}{r^4 B^6} \right]^2 - 4B^2 q^2.$$

The Figure 9 shows that all these inequalities hold all along the dynamics of the charged fluid.

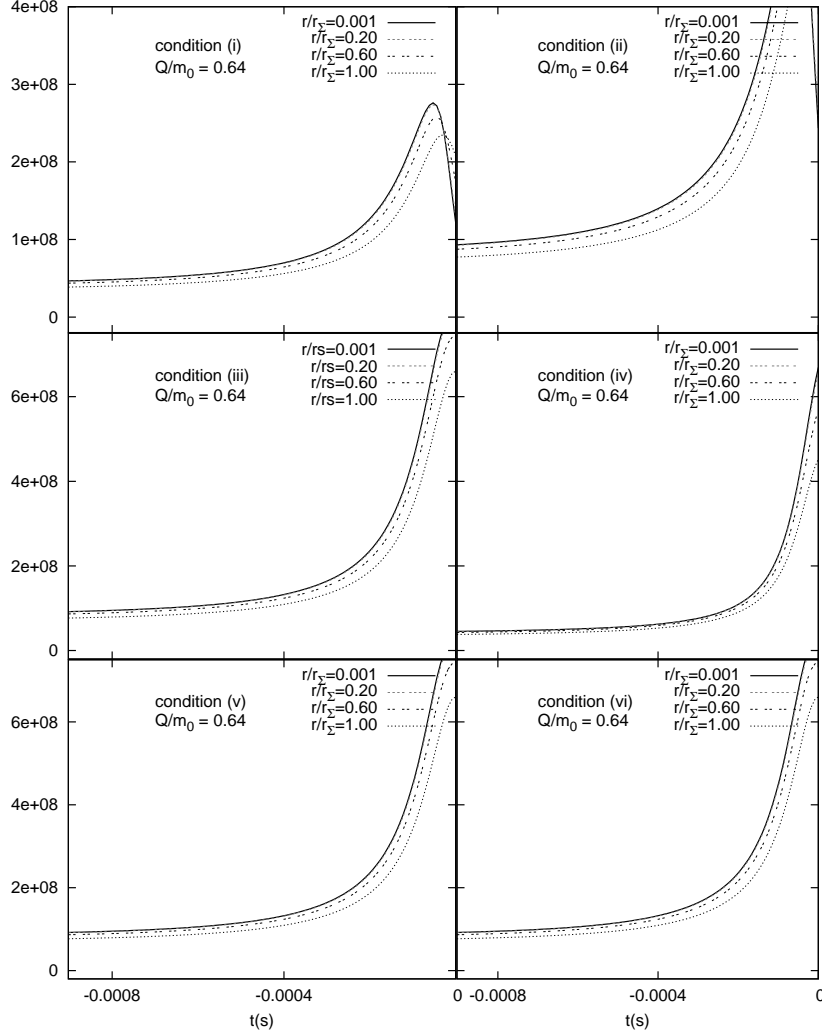


FIG. 9: Energy conditions for the highly charged model.

VII. DISCUSSION

In this work we have studied the contraction of a spherically symmetric charged body. In order to make it possible the integration of the field equations, we have to adopt some hypothesis about the interior spacetime. Firstly, its associated metric can be split into distinct functions of the radial and the temporal coordinates. Even though we can not guarantee the validity of this mathematical trick, it is a reasonable attempt to get round the underdetermination problem of the system of equations. This drives us to the second

assumption, the choice of the radial interior function. Among the uncountable possibilities of interior charged solutions, the Cooperstock and De La Cruz [8] solution was the selected one. Thus, the radial behavior of the physical variables of the system is an inheritance of such a particular choice.

In view of the previous discussion, we have concluded that appreciable physical effects to the collapse process are important for values of the net electric charge above $Q/m_0 \sim 0.10$ ($Q \sim 8.570 \times 10^{19}$ Coulomb). Furthermore, the Reissner-Nordström black hole is formed unless the total amount of charge is incredibly huge ($Q \sim 5.408 \times 10^{20}$ Coulomb for $Q/m_0 = 0.631$).

The electric field delays the event horizon formation, in agreement with the non radiative models of [5], and can even prevent the complete contraction of the body. In this case, the solution seems to point to an equilibrium situation. It is even worthy to say that, for a model with ratio $Q/m_0 \gtrsim 0.70$, neither a minimal contraction takes place.

We also have seen that, for the models that admit black hole formation ($Q/m_0 \lesssim 0.631$), those richer in charge have greater transmission of energy to the exterior region before the appearance of the event horizon, making the remnant object less massive. The luminosity increases sharply and, all of a sudden, the star turns off. Otherwise, if it is charged enough in such a way to prevent the black hole formation, the time evolution of the luminosity has a maximum peak and a subsequent reduction to zero. In contrast, the event becomes less luminous with the increasing of the fraction Q/m_0 .

Although the model presented here is, for sure, too much idealized, it certainly represents an interesting dynamical solution of Einstein's equation, and it gives us important clues on the possibility of the Reissner-Nordström black hole formation.

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